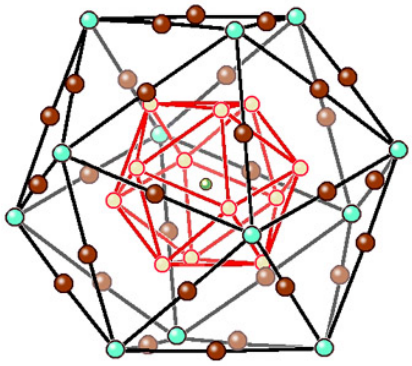
Coordinates for Mackay and Bergman clusters

icosahedron: 12 vertices cyclic permutations of $\alpha(\pm 1, \pm\tau, 0)$

docecahedron: 20 vertices cyclic permutations of $\beta(\pm 1, \pm 1, \pm 1)$ and $\beta(0, \pm\tau, \pm\sigma)$

icosidodec: 30 vertices cyclic permutations of $\gamma(\pm\sigma, \pm\tau, \pm 1)$ and $\gamma(0, \pm 2, 0)$

[$\tau = (1 + \sqrt{5})/2 \sim 1.618$, $\sigma = -1/\tau \sim -0.618$]

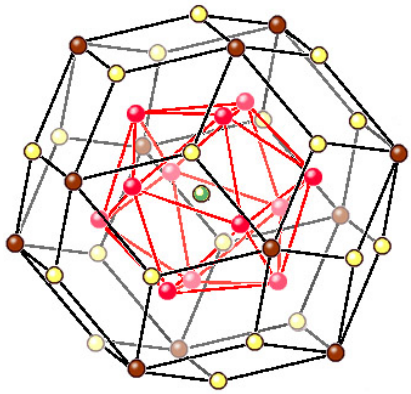


Mackay cluster

equal spheres, radius 1

First shell:	icos with	$\alpha = 1$
Second shell:	icosidodec	$\gamma = (\tau + \sqrt{3})/2 \sim 1.675$
Third shell:	icos with	$\alpha = 1 + 2/\sqrt{(2 + \tau)} \sim 2.0515$

Bergman cluster



equal spheres, radius 1

First shell:	icos with	$\alpha = 1$
Second shell:	dodec	$\beta = (\tau^2 + 2\sqrt{2})/3 \sim 1.8155$
Third shell:	icos with	$\alpha = 1 + 2/\sqrt{(2 + \tau)} \sim 2.0515$

Remarks

In both cases the spheres of the third shell touch those of the first shell but not those of the second shell.

In the Mackay case they come close: the distance between centers of a sphere in the second shell and its near neighbours in the third shell is 2.0694. The spheres of the second shell are also very close to their near neighbours in the *same* shell – distance 2.064.

In the Bergman case the gap between spheres in the second shell and near neighbours in the third shell is greater – distance 2.387. The gap is closed in an 'ideal' Bergman cluster with larger spheres in the second shell. With spheres of radius 1.433 in the second shell, each touches one sphere in the first shell and three in the third shell. In this special case, $\beta = 2.109$ for the second shell, $\beta/\alpha = 1.03$ – the 32 sphere centres of the second and third shell form an almost perfect *rhombic triacontahedron* (for which $\beta/\alpha = 1$).